Abstract—The novel decoding algorithm for the list decoding of tail-biting convolutional codes is described. The main idea is first to estimate the most reliable state from all state spaces for transmitted codeword by using a Soft Input Soft Output decoder, and then to perform the list Viterbi decoding algorithm from this state around the circular tail-biting trellis. For the LTE standard concatenated coding scheme this algorithm provides block error rate performance that is almost the same as for the optimal brute-force list decoding algorithm with significantly less complexity (about 11 times) for list size $L=4$.

I. INTRODUCTION

Concatenated coding systems are popular in many communication standards. In particular, in the LTE standard [5] the concatenated coding system consisting of an outer error detecting and an inner error correcting tail-biting convolutional code is used for transmitting short data blocks in the control and broadcasting channels. Tail-biting technique is used in convolutional codes to eliminate the rate loss caused by the known tail bits [1], that can be significant for short data blocks.

Conventionally for concatenated coding systems, the inner code is decoded first, followed by the outer code detection. Considerable improvement in performance is obtained over this conventional decoding approach when the knowledge of the $L$ best candidates to be the transmitted codeword after inner decoder is utilized during the subsequent detection [2].

Let $(n, k)$ code be a linear block code with a codelength $n$, and $k$ information symbols. The tail-biting code is a linear block code, but on the other hand it can be considered as a special (tail-biting) truncation of the convolutional code [1]. As the initial and finish state of tail-biting code is unknown in the decoder, we can not straightforwardly use the list Viterbi algorithm for decoding [2], [11] and some predecoding processing for estimating the initial state is needed.

The paper is organized as follows. Section II introduces the necessary background. A novel algorithm for the list decoding of tail-biting convolutional codes is introduced in section III. A comparison of this algorithm with the brute-force algorithm and other algorithms is presented in section IV. Finally, some conclusions are given.

II. SOME NOTIONS AND ALGORITHMS

In this section we present well-known decoding algorithms that produce a rank ordered list of the $L$ best paths through tail-biting trellis. For simplicity we consider the case of tail-biting convolutional codes with rate $R = 1/c$. Generalization to the other cases is straightforward. Let $u = (u_0, u_1, \ldots, u_{k-1})$ and $v = (v_0, v_1, \ldots, v_{k-1})$ be information and code sequences where $v_i = (v_{1i}, v_{2i}, \ldots, v_{ci})$ and let $m$ be the overall constraint length of the code [7]. This code has a circular trellis representation [3], which consists of $k$ sections. Each codeword corresponds to the circular path on this trellis.

Fig. 1. Representation of Algorithm 1 on factor graph of tail-biting convolutional code.

Existing list decoding algorithms for tail-biting convolutional codes [6], [8] are based on Algorithm 1.

Predecoding process can be based on circular Viterbi algorithm [9] like it is described in QUALCOMM patent [8] or can be performed by exhaustive search for all states from $S_0$ [6].

Simplified representation of Algorithm 1 on factor graph of tail-bitting convolutional code is given in Fig. 1, where state spaces are denoted as $S_i$, $i \in [0, k]$, and each state space consists of $|S| = |S_j| = 2^m$ states.
Algorithm 1 The list decoding algorithm for tail-biting convolutional codes.

1: Perform some predecoding process to determine the $L_s$ best candidates to be the initial state at state space $S_0$.
2: For each candidate state $s'_0$, $i \in [1, L_s]$, apply the list Viterbi decoding algorithm (LVA) for circular trellis with fixed initial and the same finish state $s'_0$ and get the list of $L_c$ codewords.
3: The final list is the best $L$ out of $L_s L_c$ codewords.

III. NOVEL DECODING ALGORITHM

The main idea of the novel decoding algorithm is to choose an initial state in some appropriate state space $S_i$ by using the Soft Input Soft Output (SISO) decoder, and apply list decoding for circular trellis with fixed initial and the same finish state from this state space $S_i$. It should be noticed that we can do it since the trellis of tail-biting convolutional code is circular. Let us first describe the novel decoding algorithm for non-recursive convolutional tail-biting codes. For non-recursive tail-biting convolutional codes the state at the moment $i$ is determined by

$$s_i = (u_{i-1} \mod k, u_{i-2} \mod k, \ldots, u_{i-m} \mod k),$$

where $s_i \in S_i$. Determine reliability of all information symbols by using some maximum-a-posteriori (MAP) decoding algorithm for tail-biting convolutional codes [4] or another algorithm for computation of reliabilities:

$$L^i_{app} = \frac{\text{Probability}(u_i = 0 | r)}{\text{Probability}(u_i = 1 | r)},$$

where $i \in [0, k - 1]$ and $r$ is received word. Estimated hard decisions of information symbols are computed as:

$$\hat{u}_i = \begin{cases} 0, & L^i_{app} \geq 0 \\ 1, & L^i_{app} < 0. \end{cases}$$

For consecutive $m$ modulo $k$ information symbols determine the measure of reliability by:

$$\Sigma_i = f([L^i_{app} - 1 \mod k], [L^i_{app} - 2 \mod k], \ldots, [L^i_{app} - m \mod k]).$$

(1)

Let us define the state space index

$$p = \arg \max_{i \in [0, k-1]} \Sigma_i$$

(2)

and

$$\hat{s}_p = ([\hat{u}_{p-1} \mod k], [\hat{u}_{p-2} \mod k], \ldots, [\hat{u}_{p-m} \mod k]).$$

(3)

The novel decoding algorithm is given in Algorithm 2.

Algorithm 2 The novel decoding algorithm for the list decoding of tail-biting convolutional codes.

1: Apply some algorithm for computation of reliability $L^i_{app}$ for all information symbols.
2: Calculate the state space index $p$ according to (2) and (1).
3: Apply the list Viterbi decoding algorithm for the circular trellis with fixed initial and the same finish state $\hat{s}_p \in S_p$, and get the list of $L$ codewords.

One possible way to choose function $f(\cdot)$ is given in the next section. Representation of Algorithm 2 on the factor graph of tail-biting convolutional code is given in Fig. 2.

Algorithm 2 is applicable for recursive systematic convolution tail-biting codes, too. In this case we first do conversion procedure [10] from recursive systematic code to non-recursive code, then perform Algorithm 2, and then do inverse conversion from non-recursive code to recursive systematic code.

IV. EXAMPLES

The LTE standard [5] contains the concatenated coding scheme consisting of outer Cyclic Redundancy Check (CRC) code with length 8 (CRC-8) and inner tail-biting convolutional code with code rate $R = 1/3$, the overall constraint length $m = 6$, and code generators (133,171,165) in octal format. Algorithm 2 can be used in that system. Let us define function $f(\cdot)$ as follows:

$$f([L^i_{app} - 1 \mod k], \ldots, [L^i_{app} - m \mod k]) = \sum_{t=i-m}^{i-1} |L^t_{app} \mod k|.$$  

(4)

In Algorithm 3 we give the application of Algorithm 2 in the LTE concatenated coding scheme.

For comparison, in Algorithms 4 and 5 we describe the Chen–Sundberg (CS) algorithm [6] and the modification of the Handlery–Johannesson–Zyablov (HZ) algorithm [12], respectively. The CS algorithm is the optimal brute-force algorithm for calculation of the $L$ best codewords. Let now compute the complexity of the novel algorithm $C_{Novel}(L)$ and the CS algorithm $C_{CS}(L)$ in terms of the number of additions and comparisons as functions of list size $L$. Firstly it should be noticed that the complexity of these algorithms is independent
from signal-to-noise ratio \((E_b/N_0)\). Define \(C_{VA}\) as the complexity required for the Viterbi decoding algorithm with fixed initial and the same finish state. Also define \(C_{LV A}(L)\) as the complexity of the List Viterbi decoding algorithm with list size \(L\) and \(C_{MLM}\) as the complexity of Max-Log-MAP decoding algorithm for tail-biting codes [4].

Thus, 
\[
C_{CS}(L) = |S| \cdot C_{VA} + L \cdot C_{LV A}(L),
\]
and \(C_{MLM}\) can be approximated as \(3 \cdot C_{VA}\). and in the case of using the PLVA as the List Viterbi decoding algorithm \(C_{LV A}(L) = L \cdot C_{VA}\). For the LTE tail-biting convolutional code \(|S| = 64\), then \(C_{CS}(L) = 64 \cdot C_{VA} + L^2 \cdot C_{VA}\) and \(C_{Novel}(L) = 3 \cdot C_{VA} + L \cdot C_{VA}\). It is easy to see that for list size \(L = 4\)
\[
\frac{C_{CS}(4)}{C_{Novel}(4)} = \frac{64 \cdot C_{VA} + 16 \cdot C_{VA}}{3 \cdot C_{VA} + 4 \cdot C_{VA}} \approx 11.4.
\]

This means that the complexity of the CS algorithm with \(L = 4\) is about 11 times more than the complexity of the novel algorithm 3 for the LTE tail-biting convolutional code. The HJZ algorithm has less complexity than the novel decoding algorithm but provides less Block error rate (BLER) performance.

Block error rate (BLER) performance dependence on signal-to-noise ratio \((E_b/N_0)\) for additive white Gaussian noise (AWGN) channel and QPSK modulation of the novel decoding algorithm, CS algorithm [6], HJZ algorithm [12], and the QUALCOMM algorithm [8] with almost the same complexity as the novel decoding algorithms are given in Fig. 3, 4. The concatenated codes are (120,32) and (180,52) codes. respectively. List size of all compared algorithms is chosen as \(L = 4\).

From Fig. 3 we see that the difference between the novel algorithm and the CS algorithm is about 0.1 dB, and the novel algorithm outperforms HJZ and QUALCOMM algorithms by 0.5 dB and 0.2 dB respectively at level of \(10^{-2}\) of BLER. From Fig. 4 we see that there is no difference between the novel algorithm and the CS algorithm, and the novel algorithm outperforms HJZ and QUALCOMM algorithms by 0.5 dB and 0.2 dB respectively at level of \(10^{-2}\) of BLER.

There are also a few decoding algorithms of tail-biting codes [13], [14], [15] that also could be applied to list decoding. But their complexities strongly depend on signal-to-noise ratio in the channel.

V. CONCLUSION

In this paper the novel decoding algorithm for the list decoding of tail-biting convolutional codes was proposed. The main idea is first to estimate the most reliable state from all state spaces for transmitted codeword by using the SISO decoder, and then to perform the list Viterbi decoding algorithm and the CS algorithm is about 0.1 dB, and the novel algorithm outperforms HJZ and QUALCOMM algorithms by 0.5 dB and 0.2 dB respectively at level of \(10^{-2}\) of BLER.

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There are also a few decoding algorithms of tail-biting codes [13], [14], [15] that also could be applied to list decoding. But their complexities strongly depend on signal-to-noise ratio in the channel.
algorithm from this state around circular tail-biting trellis. Concatenated coding system consisting of CRC and tail-biting convolutional code from the LTE standard is used for simulation. Block error rate performance of the novel decoder was compared with other decoders for short and medium code lengths over AWGN channel and QPSK modulation. It was shown that performance of the novel decoder is almost the same as performance of optimal brute force decoder with significantly less complexity (about 11 times). Complexity of novel algorithm is independent of signal-to-noise ratio in the channel.

REFERENCES